Sketch

Simple 3D sketching

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1 About sketch

`Sketch` is a small, simple system for producing line drawings of two- or three-dimensional objects and scenes. It began as a way to make illustrations for a textbook after we could find no suitable tool for this purpose. Existing scene processors emphasized GUIs and/or photo-realism, both un-useful to us. We wanted to produce finely wrought, mathematically-based illustrations with no extraneous detail.

`Sketch` accepts a tiny scene description language and generates `PSTricks` or `TikZ/PGF` code for LaTeX. The `sketch` language is similar to `PSTricks`, making it easy to learn for current `PSTricks` users. See `www.pstricks.de` for information on `PSTricks`. `TikZ/PGF` are also very similar except for details of syntax. See `http://sourceforge.net/projects/pgf`. One can easily lay raw `PSTricks` or `TikZ/PGF` output over, in, or under `sketch` drawings, providing the full power of LaTeX text and mathematics formatting in a three-dimensional setting.

1.1 Reporting bugs and recommending improvements.

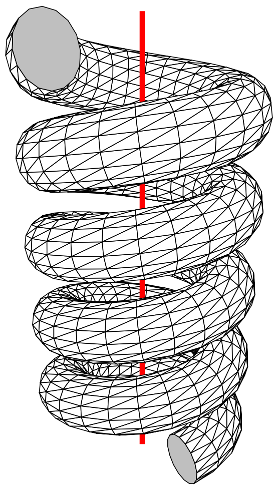
Send bug reports and suggestions to `sketch@frontiernet.net`. We will try to respond, but can't promise. In any event, don't be offended if a reply is not forthcoming. We're just busy and will get to your suggestion eventually.

For bugs, attach a `sketch` input file that causes the bad behavior. Embed comments that explain what to look for in the behavior of `sketch` or its output.

A recommendation for improvement from one unknown person counts as one vote. We use overall vote tallies to decide what to do next as resources permit. We reserve the right to a assign any number of votes to suggestions from people who have been helpful and supportive in the past.

1.2 Contributions

If you intend to implement an enhancement of your own, that's terrific! Consider collaborating with us first to see if we're already working on your idea or if we can use your work in the official release.

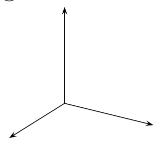


2 Introduction by example

The sketch input language will seem familiar to users of the PSTricks package for LaTeX. The following program draws a triangular polygon pierced by a line.

```
polygon(0,0,1)(1,0,0)(0,1,0)
line(-1,-1,-1)(2,2,2)
```

The coordinate system is a standard right-handed Cartesian one.



2.1 Hello world

The sketch program above is nearly the simplest one possible, the equivalent of a “hello world” program you might find at the start of a programming language text. If it is saved in the file ‘simple.sk’, then the command

```
sketch simple.sk -o simple.tex
```

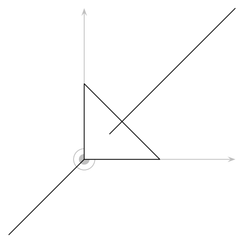
creates a file ‘simple.tex’ containing PSTricks commands to draw these objects on paper. The contents of ‘simple.tex’ look like this.

```
\begin{pspicture}(-1,-1)(2,2)
\pstVerb{1 setlinejoin}
\psline(-1,-1)(.333,.333)
\pspolygon[fillstyle=solid,fillcolor=white](0,0)(1,0)(0,1)
\psline(.333,.333)(2,2)
\end{pspicture}
```

The hidden surface algorithm of sketch has split the line into two pieces and ordered the three resulting objects so that the correct portion of the line is hidden.

If you’ve noticed that the projection we are using seems equivalent to erasing the z-coordinate of the three-dimensional input points, pat yourself on the back. You are correct. This is called a *parallel projection*. The z-coordinate axis is pointing straight out of the paper at us, while the x- and y-axes point to the right and up as usual.

The resulting picture file can be included in a LaTeX document with \input{simple}. Alternately, adding the command line option ‘-T’[1] causes the pspicture to be wrapped in a short but complete document, ready to run though LaTeX. In a finished, typeset document, the picture looks like this. (The axes have been added in light gray.)



[1] Or for European users of A4 size paper, ‘-Te’.

It is important to know that only the “outside” of a polygon is normally drawn. The *outside* is where the vertices given in the `polygon` command appear in *counter-clockwise* order. Thus, if the command above had been

```
polygon(0,1,0)(1,0,0)(0,0,1)
```

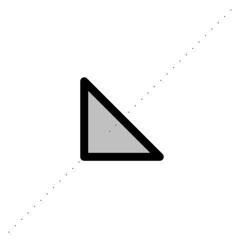
the polygon would not appear in the picture at all. It would have been *culled* from the scene. This culling behavior may seem strange, but stay tuned.

2.2 Options

Many `PSTricks` and `TikZ/PGF` options work just fine in `sketch`. If generating `PSTricks`, the code

```
polygon[fillcolor=lightgray,linewidth=3pt](0,0,1)(1,0,0)(0,1,0)
line[linestyle=dotted](-1,-1,-1)(2,2,2)
```

produces



To produce `TikZ/PGF`, the corresponding code is

```
polygon[fill=lightgray,line width=3pt](0,0,1)(1,0,0)(0,1,0)
line[style=dotted](-1,-1,-1)(2,2,2)
global { language tikz }
```

The final `global` instructs `sketch` to produce `TikZ/PGF` code as output rather than the default, `PSTricks`. Note that `polygon` fill color and `line` style options both conform to `TikZ` syntax rules. The remaining examples of this manual are in PSTricks style.

2.3 Drawing a solid

Let’s try something more exciting. `Sketch` has no notion of a solid, but polygonal *faces* can be used to represent the boundary of a solid. To the previous example, let’s add three more triangular polygons to make the faces of an irregular tetrahedron.

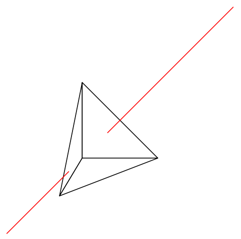
```
% vertices of the tetrahedron
def p1 (0,0,1) def p2 (1,0,0)
def p3 (0,1,0) def p4 (-.3,-.5,-.8)

% faces of the tetrahedron.
polygon(p1)(p2)(p3) % original front polygon
polygon(p1)(p4)(p2) % bottom
polygon(p1)(p3)(p4) % left
polygon(p3)(p2)(p4) % rear

% line to pierce the tetrahedron
line[linecolor=red](-1,-1,-1)(2,2,2)
```

This example uses *definitions*, which begin with `def`. These *define* or give names to points, which are then available as *references* by enclosing the names in parentheses, e.g. `(foo)`. The parentheses denote that the names refer to points; they are required. There can be no white space between them and the name.

As you can see, comments start with `%` as in TEX and extend to the end of the line (though `#` will work as well). White space, including spaces, tabs and blank lines, has no effect in the `sketch` language.

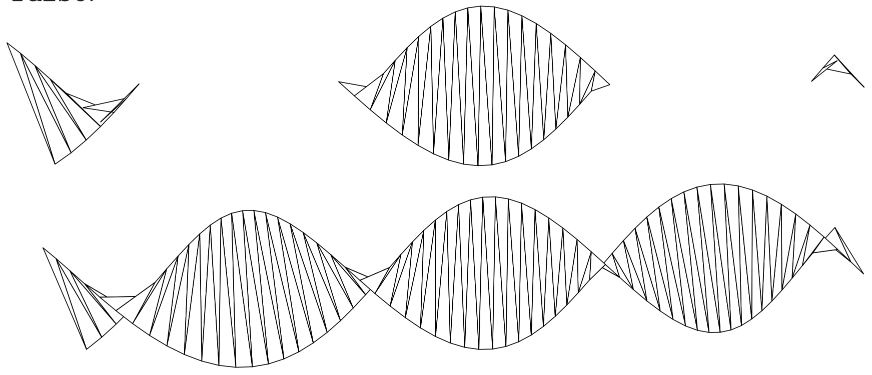


If we look inside the TEX file produced by `sketch`, there will be only three polygons. The fourth has been culled because it is a "back face" of the tetrahedron, invisible to our view. It is unnecessary, and so it is removed.

In some drawings, polygons act as zero-thickness solid surfaces with both sides visible rather than as the faces of solid objects, where back faces can be culled. For zero-thickness solids, culling is a problem. One solution is to use a pair of `sketch` polygons for each zero-thickness face, identical except with opposite vertex orders. This is unwieldy and expensive. A better way is to set the `sketch` internal option `cull` to `false` in the usual `PSTricks` manner.

```
polygon[cull=false](p1)(p2)(p3)
```

The following shows the same helix shape drawn first with `cull=true` (the default) and then `cull=false`.



We'll soon see how to produce these helixes with a few lines of `sketch` language code.

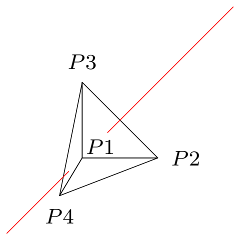
It may be tempting to turn culling off gratuitously so that vertex order can be ignored. This is not a good idea because output file size and TEX and Postscript processing time both depend on the number of output polygons. Culling usually improves performance by a factor of two. On the other hand, globally setting `cull=false` is reasonable while debugging. See Section 3.4.1 [Global options], page 22 and Section 4.4.1 [Limits on error detection], page 33.

2.4 Special objects

We can add labels to a drawing by using `special` objects, which provide a way to embed raw LaTeX and `PSTricks` code. Adding this to the tetrahedron does the trick.

```
special |\footnotesize
         \uput{2pt}[ur]#1{$P1$}
         \uput[r]#2{$P2$}
         \uput[u]#3{$P3$}
         \uput[d]#4{$P4$}|
  (p1)(p2)(p3)(p4)
```

Here is the result.



There are several details to note here. First, the quoting convention for the raw code is similar to the LaTeX `\verb` command. The first non-white space character following `special` is understood to be the quote character, in this case '`|`'. The raw text continues until this character recurs.

Second, the argument references `#1`, `#2`, `#3`, and `#4` refer to points in the list that follow. This is similar to TeX macro syntax. The transformed and two-dimensional projections of these three-dimensional points are substituted in the final output. An argument reference of the form `#1-2` is replaced with the angle in degrees of the two-dimensional vector that connects the projections of the two respective argument points, here `#1` and `#2`. The substituted angle is enclosed in curly braces `{ }`

By default, `special` objects are printed last, overlaying all other objects in the scene. If you specify the internal option `lay=in`, the hidden surface algorithm considers the entire special object to be the first point (`#1`) in the argument list. If that point is behind (of smaller z-component than) any drawable, then the entire special object is drawn before that drawable, so the drawable obscures parts of the special object that overlaps it. In our example, `p1` is the front-most point in the scene (has the largest z-component), so adding `lay=in` has no effect.

With option `lay=under`, a special is drawn *before*, hence appears *under* any of the objects handled by the hidden surface algorithm. This is how the light gray axes were added to the "hello world" example Section 2.1 [Hello world], page 2.

`Special` objects are powerful, with many possible uses.

2.5 Transforms

Now let's add a second copy of the pierced tetrahedron. We'll rotate the copy 90 degrees about the x-axis with the origin as *center of rotation* so we can see the back, then translate it to the right—in the positive x-direction—so it doesn't collide with the original. To help us see what's going on, make the back side gray.

```
def pierced_tetrahedron {
  def p1 (0,0,1) def p2 (1,0,0)
  def p3 (0,1,0) def p4 (-.3,-.5,-.8)
  polygon(p1)(p2)(p3)                      % original
  polygon(p1)(p4)(p2)                      % bottom
  polygon(p1)(p3)(p4)                      % left
  polygon[fillcolor=lightgray](p3)(p2)(p4) % rear
  line[linecolor=red](-1,-1,-1)(2,2,2)
}
{pierced_tetrahedron}  % tetrahedron in original position
put { rotate(90, (0,0,0), [1,0,0]) % copy in new position
      then translate([2.5,0,0]) } {pierced_tetrahedron}
```

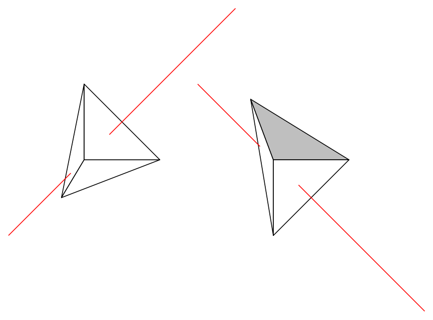
Here the entire code of the previous example has been wrapped in a definition by forming a *block* with braces (a single item would not need them). The point definitions nested inside the braces are *lexically scoped*. Their meaning extends only to the end of the block. The outer `def` is called a *drawable* definition because it describes something that can be drawn.

A drawable definition by itself causes nothing to happen until its name is referenced. Drawable references must be enclosed in curly braces, e.g. `{foo}`, with no intervening white space. In the code above, the first reference `{pierced_tetrahedron}` is a plain one. Its effect is merely to duplicate the earlier drawing. Almost any series of `sketch` commands `stuff` may be replaced with `def foo { stuff } {foo}` without changing its meaning.

The `put` command supplies a second reference, this time with a *transform* applied first. The `rotate` transform turns the tetrahedron 90 degrees about the origin. The axis of rotation is the vector $[1, 0, 0]$. By the *right hand rule*, this causes the top of the tetrahedron to rotate toward the viewer and the bottom away. The rule receives its name from the following definition:

> **Right hand rule.** If the right hand is wrapped around any axis with the thumb pointing in the direction, then the fingers curl in the direction of positive rotation about that axis.

The `translate` transform moves the pyramid laterally to the right by adding the vector $[2.5, 0, 0]$ to each vertex coordinate. The result is shown here.



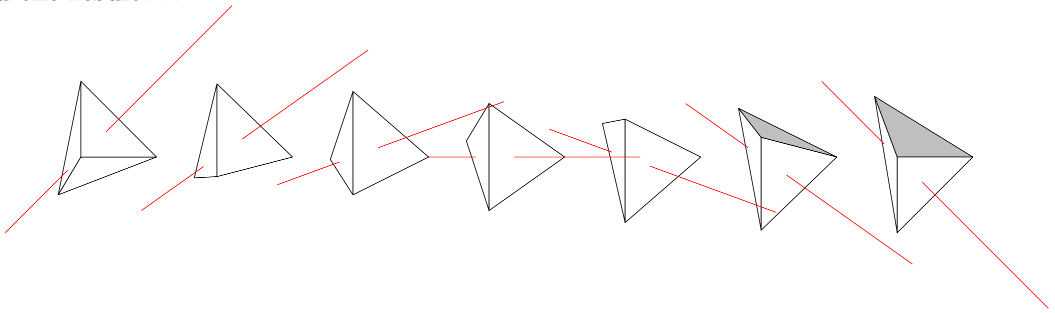
2.6 Repeated objects

To draw seven instances of the tetrahedron, each differing from the last by the same transform, replace the last two commands of the previous example with

```
repeat { 7, rotate(15, (0,0,0), [1,0,0]) % copy in new position
```

```
                then translate([2,0,0]) } {pierced_tetrahedron}
```

And the result. . .



2.7 Swept objects

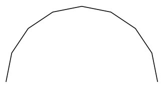
Many familiar shapes can be generated by sweeping simpler ones through space and considering the resulting path, surface, or volume. `Sketch` implements this idea in the `sweep` command.

```
def n_segs 8
sweep { n_segs, rotate(180 / n_segs, (0,0,0), [0,0,1]) } (1,0,0)
```

This code sweeps the point $(1, 0, 0)$ eight times by rotating it $180/8 = 22.5$ degrees each time and connecting the resulting points with line segments. The `def` used here is a *scalar* definition. References to scalars have no enclosing brackets at all.

2.7.1 Point sweeps

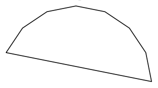
Sweeping a point makes a one-dimensional path, which is a polyline. Since we have swept with a rotation, the result is a circular arc. Here is what it looks like.



This is the first example we have seen of `sketch` arithmetic. The expression `180 / n_segs` causes the eight rotations to add to 180. If you're paying attention, you'll have already noted that there are *nine* points, producing eight line segments.

You can cause the swept point to generate a single polygon rather than a polyline by using the *closure tag* `<>` after the number of swept objects. Code and result follow

```
def n_segs 8
sweep { n_segs<>, rotate(180 / n_segs, (0,0,0), [0,0,1]) } (1,0,0)
```



2.7.2 Polyline sweeps

Sweeping a polyline produces a surface composed of many faces. The unbroken helix in the example [Helix with cull set false then true], page 4 is produced by this code (plus a surrounding `put` rotation to make an interesting view; this has been omitted).

```
def K [0,0,1]
sweep[cull=false] {
  60,
  rotate(10, (0,0,0), [K]) then translate(1/6 * [K])
} line[linewidth=2pt](-1,0)(1,0)
```

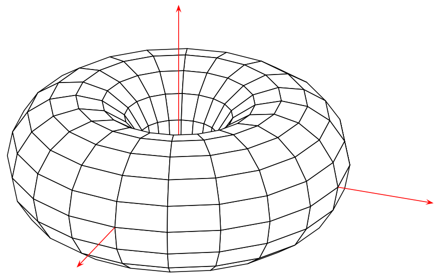
Again, 60 segments of the helix are produced by connecting 61 instances of the swept line. Options applied to the sweep, here `cull=false`, are treated as options for the generated polygon or polyline. Options of the swept line itself, here `linewidth=2pt`, are ignored, though with a warning. This `def` is a *vector* definition, which must be referenced with square brackets, e.g. `[foo]`.

2.7.3 Nested sweeps

When the center point of rotation is omitted, the origin is assumed. When a point has only two coordinates, they are taken as x and y, with $z = 0$ assumed. A toroid is therefore obtained with this code.

```
  def n_toroid_segs 20   def n_circle_segs 16
  def r_minor 1          def r_major 1.5
  sweep { n_toroid_segs, rotate(360 / n_toroid_segs, [0,1,0]) }
    sweep { n_circle_segs, rotate(360 / n_circle_segs, (r_major,0,0)) }
      (r_major + r_minor, 0)
```

For intuition, the idea of the code is to sketch a circle to the right of the origin in the xy-plane, then rotate that circle “out of the plane” about the y-axis to make the final figure. This produces the following. (A view rotation and some axes have been added.)



This example also shows that the swept object may itself be another `sweep`. In fact, it may be *any* `sketch` expression that results in a list of one or more points or, alternately, a list of one or more polylines and polygons. The latter kind of list can be created with a `{ }`-enclosed block, perhaps following a `put` or `repeat`.

2.7.4 Polygon sweeps

Sweeping a polygon creates a closed surface with polygons at the ends, which are just copies of the original, appropriately positioned. See [Solid coil example], page 1. Options on the swept polygon, if they exist, are applied to the ends. Otherwise the sweep options are used throughout.

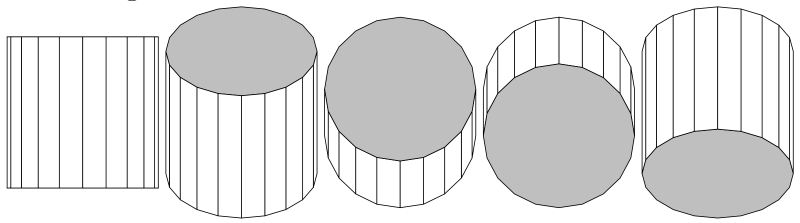
2.7.5 Polyline sweeps with closure

A polyline sweep with a closure tag creates another kind of closed surface. First, the polyline segments are connected by faces, just as without the closure tag. Then, each set of end points is joined to make a polygon, one for each end. A code for several views of a cylindrical prism follows.

```
  def n_cyl_segs 20  def n_views 5  def I [1,0,0]
```

```
def endopts [fillcolor=lightgray]
repeat { n_views, rotate(180/n_views, [I]) then translate([I] * 2.1) }
  sweep[endopts]{ n_cyl_segs<>, rotate(360/n_cyl_segs, [0,1,0]) }
    line[fillcolor=white](1,-1)(1,1)
```

It produces this drawing.



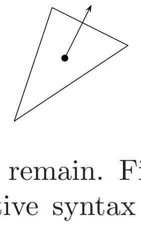
The options of the swept line, if any, are applied to the faces produced by sweeping the line, but not the end polygons. Otherwise, the sweep options are applied throughout. The `def` in this example is an *option* definition. References to options must be enclosed in square brackets, e.g. `[foo]`. Happily, the syntax of `sketch` is such that options references can never be confused with vector references. While not apparent in this example, options references are useful when defining many objects with a similar appearance.

2.7.6 Affine arithmetic

The arithmetic `[I] * 2.1` above hints at a larger truth. `Sketch` operators work on scalars, vectors, points, and transforms according to the general rules of *affine algebra*. This can be helpful for setting up diagrams with computed geometry. For example, if you have triangle vertices `(p1)` through `(p3)` and need to draw a unit normal vector pointing out of the center of the triangle, this code does the trick.

```
def p1 (1,0,0)  def p2 (0,0.5,0)  def p3 (-0.5,-1,2)
def O (0,0,0)
def N unit( ((p3) - (p2)) * ((p1) - (p2)) )
def n1 ((p1)-(O) + (p2)-(O) + (p3)-(O)) / 3 + (O)
def n2 (n1)+[N]
polygon(p1)(p2)(p3)
line[arrows=*->](n1)(n2)
```

The first line computes the cross product of two edge vectors of the triangle and scales it to unit length. The second computes the average of the vertices. Note that subtraction and addition of the origin effectively convert vectors to points and *vice versa*. The line command draws the normal at the correct spot.



Two caveats regarding this example remain. First, the only way to use `PSTricks`-style arrows is with `arrows=`. The alternative syntax for `PSTricks` arrows is not allowed in `sketch`. Second, you might like to eliminate the third `def` and write instead the following.

```
line[arrows=*->](n1) (n1)+[N]
```

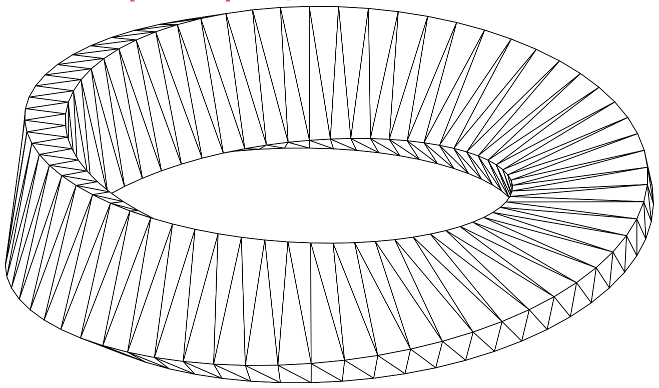
This is not allowed. The point lists in drawables may consist only of explicit points or point references. You may, however, use arithmetic to calculate point components. The following works, though it's a little cumbersome.

```
  line[arrows=*->](n1)((n1)'x+(N)'x, (n1)'y+(N)'y, (n1)'z+(N)'z)
```

Obviously, the *tick operator* '`'x`' extracts components of points and vectors.

2.7.7 More to learn

This is not the end of the story on sweeps! We invite the reader into the main body of this documentation Section 3.2.6 [Sweeps], page 18 to learn more.



Who knows where you'll finish?

3 Input language

This chapter describes the `sketch` input language in detail.

3.1 Basics

`Sketch` input is plain ASCII text, usually stored in an input file. It describes a *scene*, so the sketch language is a *scene description language*. `Sketch` input is also *declarative*. It merely declares what the scene ought to look like when drawing is complete and says very little about how `sketch` should do its work. `Sketch` commands are not executed sequentially as in the usual programming language. They merely contribute to that declaration.

A few syntactic details are important. Case is significant in the `sketch` language. With a few exceptions, white space is not. This includes line breaks. Comments begin with `%` or `#` and extend to the end of the line. You can disable a chunk of syntactically correct `sketch` code by enclosing it in a `def`. There is a simple "include file" mechanism. The command

```
input{otherfile.sk}
```

causes the contents of '`otherfile.sk`' to be inserted as though they were part of the current file.

3.1.1 Identifiers

Identifiers in `sketch` are references to earlier-defined options, scalars, points, vectors, transforms, drawables, and tags. Definitions are explained in Section 3.3 [Definitions], page 21.

An identifier consists of a leading letter followed by letters, numbers and underscores. The last character may *not* be an underscore. Keywords cannot be used as identifiers, and reserved words ought to be avoided. See Section 3.1.2 [Key and reserved words], page 11.

3.1.2 Key and reserved words

The keywords of `sketch` are `picturebox curve def dots frame global input line polygon put repeat set sweep` and `then`. The `sketch` parser will note a syntax error if any of these are used in place of a proper identifier.

In addition, there are reserved words that can currently be defined by the user, but with the risk that future versions of `sketch` will reject those definitions. The reserved words are `atan2 cos inverse perspective project rotate scale sin special sqrt translate unit` and `view`.

3.1.3 Literals

Literals in `sketch` include scalars, points, vectors, and transforms. Literals, along with defined object references, are used in arithmetic expressions. See Section 3.1.4 [Arithmetic], page 13.

3.1.3.1 Scalar literals

Scalar literals are positive floating point numbers with syntax according to C conventions. The following are some examples.

```
0 1004 .001 8.3143 3. 1.60E-19 6.02e+23
```

Scalar literals may not contain embedded spaces.

3.1.3.2 Point and vector literals

Points and vector literals have these forms respectively.

```
(X,Y,Z)  [X,Y,Z]
```

Each of the components is itself a scalar expression. The z-components are optional and default to zero.

3.1.3.3 Transform literals

Most transform literals are formed by *constructors*. These are summarized in the following table.

Constructor	**Param types**	**Description**
`rotate(A,P,X)`	scalar,point,vector	Rotate `A` degrees about point `P` with axis `X` according to the right hand rule. See [Right hand rule], page 6. `P` and `X` are both optional and default to the origin and the z-axis respectively.
`translate(X)`	vector	Translate by `X`.
`scale(S)`	scalar	Scale uniformly by factor `S`.
`scale(V)`	vector	Scale along each axis by components of `V`.
`project()`	—	Same as `scale([1,1,0])`.
`project(S)`	scalar	Perspective projection with view center at origin and projection plane $z = -$`S`.
`perspective(S)`	scalar	Perspective *transform* identical to `project(S)` except that the z-coordinate of the transformed result is *pseudodepth*, usable by the hidden surface algorithm.
`view(E,D,U)`	point,vector,vector	View transform similar to that of `OpenGL`'s. The *eye point* `E` is translated to the origin while a rotation is also applied that makes the *view direction vector* `D` and the *view "up" vector* `U` point in the negative z- and the y-directions respectively. If `U` is omitted, it defaults to $[0, 1, 0]$. When `U` is omitted, `D` may be also; it defaults to `(0,0,0)-(E)`, a vector pointing from the eye toward the origin.
`view(E,L,U)`	point,point,vector	An alternate form of `view(E,D,U)` above where the view direction parameter `D` is replaced with a "look at" point `L`, i.e., a point where the viewer is focusing her attention. This form of view is equivalent to `view(E, (L)-(E), U)`, where `(L)-(E)` is a direction vector. `U` is optional and defaults to $[0, 1, 0]$.
`[[`a_{11}`,`a_{12}`,`a_{13}`,`a_{14}`]` `[`a_{21}`,`a_{22}`,`a_{23}`,`a_{24}`]` `[`a_{31}`,`a_{32}`,`a_{33}`,`a_{34}`]` `[`a_{41}`,`a_{42}`,`a_{43}`,`a_{44}`]]`	16 scalars	Direct transform matrix definition. Each of the a_{ij} is a scalar expression. If you don't know what this is about, you don't need it.

The `project` constructor is not generally useful because it defeats hidden surface removal by collapsing the scene onto a single plane. It is a special purpose transform for drawing pictures of scenes where three-dimensional objects are being projected onto planes. See, for example, Section 4.1 [Overview], page 24.

3.1.4 Arithmetic expressions

Arithmetic expressions over `sketch` literals and defined identifiers are summarized in the following tables.

3.1.4.1 Two-operand (binary) forms and precedence

Most two-operand binary forms have meanings dependent on the types of their arguments. An exhaustive summary of the possibilities is given in the following table.

Left	**Op**	**Right**	**Result**	**Description**
scalar	`+`	scalar	scalar	Scalar sum.
vector	`+`	vector	vector	Vector sum.
point	`+`	vector	point	Point-vector affine sum.
vector	`+`	point	"	"
scalar	`-`	scalar	scalar	Scalar difference.
vector	`-`	vector	vector	Vector difference.
point	`-`	point	vector	Point-point affine difference.
point	`-`	vector	point	Point-vector affine difference.
scalar	`*` or `.`	scalar	scalar	Scalar product.
scalar	`*` or `.`	vector	vector	Scalar-vector product.
vector	`*` or `.`	scalar	"	"
vector	`*`	vector	vector	Vector cross-product.
vector	`.`	vector	scalar	Vector dot product.
scalar	`^`	scalar	scalar	Raise scalar to scalar power.
transform	`^`	integer	transform	Raise transform or integer power.
transform	`*` or `.`	point	point	Affine point transform (right-to-left).
transform	`*` or `.`	vector	vector	Affine vector transform (right-to-left).
transform	`*` or `.`	transform	transform	Transform composition (right-to-left).
point	`then`	transform	point	Affine point transform (left-to-right).
vector	`then`	transform	vector	Affine vector transform (left-to-right).
transform	`then`	transform	transform	Transform composition (left-to-right).
scalar	`/`	scalar	scalar	Scalar division.
vector	`/`	scalar	vector	Vector component-wise division by scalar.
point	`'`	`x`, `y`, or `z`	scalar	Point component extraction.
vector	`'`	`x`, `y`, or `z`	scalar	Vector component extraction.

Operator precedence is shown in this table.

Op	**Precedence**
`'`	highest (most tightly binding)
`^`	
`-`	(unary negation)

`* . /`	
`+ -`	
`then`	lowest (least tightly binding)

All operations are left-associative except for '`^`'. Parentheses '`( )`' are used for grouping to override precedence in the usual way.

As you can see, the dot operator '`.`' is usually a synonym for run-of-the-mill multiplication, '`*`'. The meanings differ only for vector operands. The `then` operator merely reverses the operand order with respect to normal multiplication '`*`'. The intent here is to make compositions read more naturally. The code

```
(1,2,3) then scale(2) then rotate(30) then translate([1,3,0])
```

expresses a series of successive modifications to the point, whereas the equivalent form

```
translate([1,3,0]) * rotate(30) * scale(2) * (1,2,3)
```

will be intuitive only to mathematicians (and perhaps Arabic language readers).

3.1.4.2 Unary forms

Unary or one-operand forms are summarized in the following table, where `X` stands for the operand.

Op	**Operand**	**Result**	**Description**
`-X`	scalar	scalar	Unary scalar negation.
`-X`	vector	vector	Unary vector negation.
`|X|`	vector	scalar	Vector length.
`unit(X)`	vector	vector	Unit vector with same direction.
`sqrt(X)`	scalar	scalar	Scalar square root.
`sin(X)`	scalar	scalar	Trigonometric sine (`X` in degrees).
`cos(X)`	scalar	scalar	Trigonometric cosine (`X` in degrees).
`atan2(X,Y)`	scalar	scalar	Polar angle in degrees of vector $[X, Y]$.
`inverse(X)`	transform	transform	Inverse transform.

Errors are reported when `|X|`, `unit`, `sqrt`, `atan2`, and `inverse` fail due to bad parameters.

3.1.5 Options

Syntax:

```
[key1=val1,key2=val2,...]
```

Options are used to specify details of the appearance of drawables. As shown above, they are given as comma-separated key-value pairs.

3.1.5.1 PSTricks options

When `language pstricks` is selected (the default), permissible key-value pairs include all those for similar `PSTricks` objects. For example, a polygon might have the options

```
[linewidth=1pt,linecolor=blue,fillcolor=cyan]
```

`Sketch` merely passes these on to `PSTricks` without checking or modification. Option lists are always optional. A missing options list is equivalent to an empty one '`[]`'.

When a `polygon` has options for both its face and its edges, and the polygon is split by the hidden surface algorithm, `sketch` must copy the edge options to `pslines` for the edge segments and the face options to `pspolygons`. Options known to `sketch` for purposes of this splitting operation include `arrows`, `dash`, `dotsep`, `fillcolor`, `fillstyle`, `linecolor`, `linestyle`, `linewidth`, `opacity`, `showpoints`, `strokeopacity`, and `transpalpha`.

3.1.5.2 `TikZ/PGF` options

`TikZ/PGF` options are handled much as for `PSTricks`. Though `TikZ/PGF` often allows colors and styles to be given without corresponding keys, for example,

```
\draw[red,ultra thick](0,0)--(1,1);
```

this is not permitted in `sketch`. To draw a red, ultra-thick line in `sketch`, the form is

```
line[draw=red,style=ultra thick](0,0)(1,1)
```

Just as for `PSTricks`, when a `polygon` has options for both its face and its edges, and the polygon is split by the hidden surface algorithm, `sketch` must copy the edge options to `pslines` for the edge segments and the face options to `pspolygons`. `TikZ/PGF` options known to `sketch` for purposes of this splitting operation include `arrows`, `cap`, `color`, `dash pattern`, `dash phase`, `double distance`, `draw`, `draw opacity`, `fill`, `fill opacity`, `join`, `line width`, `miter limit`, `pattern`, `pattern color`, and `style`.

The `style` option can contain both face and edge information, so `sketch` must check the style value. Values known to `sketch` include `dashed`, `densely dashed`, `densely dotted`, `dotted`, `double`, `loosely dashed`, `loosely dotted`, `nearly opaque`, `nearly transparent`, `semithick`, `semitransparent`, `solid`, `thick`, `thin`, `transparent`, `ultra nearly transparent`, `ultra thick`, `ultra thin`, `very nearly transparent`, `very thick`, and `very thin`.

3.1.5.3 Dots in `TikZ/PGF`

`TikZ/PGF` does not have a `dots` command as does PSTricks. Instead, `Sketch` emits dots as `filldraw` circles. The diameter may be set using the option `dotsize` borrowed from PSTricks. The `dotsize` option will be removed from the option list in the output `filldraw` command. Other options work in the expected way. For example, `fill` sets fill color and `color` sets line color of the circles.

3.1.5.4 `TikZ/PGF` user-defined styles

`TikZ/PGF` allows named styles defined by the user, for example

```
\tikzstyle{mypolygonstyle} = [fill=blue!20,fill opacity=0.8]
\tikzstyle{mylinestyle} = [red!20,dashed]
```

Since `sketch` has no information on the contents of such styles, it omits them entirely from lines, polygons, and their edges during option splitting. For example,

```
polygon[style=mypolygonstyle,style=thick](0,0,1)(1,0,0)(0,1,0)
line[style=mylinestyle](-1,-1,-1)(2,2,2)
```

produces the `TikZ` output

```
\draw(-1,-1)--(.333,.333);
\filldraw[thick,fill=white](0,0)--(1,0)--(0,1)--cycle;
\draw(.333,.333)--(2,2);
```

Note that the user-defined styles are not present. Sketch also issues warnings:

```
warning, unknown polygon option style=mypolygonstyle will be ignored
warning, unknown line option style=mylinestyle will be ignored
```

The remedy is to state explicitly whether a user-defined style should be attched to polygons or lines in the TikZ output using *pseudo-options* `fill style` and `line style`,

```
polygon[fill style=mypolygonstyle,style=thick](0,0,1)(1,0,0)(0,1,0)
line[line style=mylinestyle](-1,-1,-1)(2,2,2)
```

Now, the output is

```
\draw[mylinestyle](-1,-1)--(.333,.333);
\filldraw[mypolygonstyle,thick](0,0)--(1,0)--(0,1)--cycle;
\draw[mylinestyle](.333,.333)--(2,2);
```

A useful technique is to include user-defined style definitions in `sketch` code as `specials` with option `[lay=under]` to ensure that the styles are emitted first in the output, before any uses of the style names.[1] For example,

```
special|\tikzstyle{mypolygonstyle} = [fill=blue!20,fill opacity=0.8]|[lay=under]
special|\tikzstyle{mylinestyle} = [red!20,dashed]|[lay=under]
```

The author is responsible for using the key, `line style` or `fill style`, that matches the content of the style definition.

3.1.5.5 Transparency

Both `PSTricks` and `TikZ/PGF` support polygon options that have the effect of making the polygon appear transparent. For `PSTricks`, keywords `opacity` and `transpalpha` have both been used, with the correct one depending on version. `TikZ/PGF` uses `opacity` only. When transparent polygons are in the foreground, objects behind them (drawn earlier) are visible with color subdued and tinted. The hidden surface algorithm of `sketch` works well with such transparent polygons.

Note that `cull=false` must be used for rear-facing polygons to be visible when positioned behind other transparent surfaces.

3.1.5.6 Internal options

There are also *internal* options used only by `sketch` and not passed on to `PSTricks`. These are summarized in the following table.

Key	**Possible values**	**Description**
`cull`	`true`, `false`	Turn culling of backfaces on and off respectively for this object. The default value is `true`.
`lay`	`over`, `in`, `under`	Force this object to be `under` or `over` all other objects in the depth sort order created by the hidden surface algorithm. The default value `over` guarantees that output due to the `special` will be visible.
`split`	`true`, `false`	Turn splitting of sweep-generated body polygons on and off respectively. See Section 3.2.6 [Sweeps], page 18. The default value `true` causes “warped” polygons to be split into triangles, which avoids mistakes by the hidden surface algorithm.

[1] This clever trick is due to Kjell Magne Fauske.

3.1.6 Point lists

Syntax:

```
(x1,y1,z1)(x2,y2,z2)...
```

A sequence of one or more points makes a point list, a feature common to all drawables. Each of the point components is a scalar arithmetic expression. Any point may have the z-component omitted; it will default to $z = 0$.

3.2 Drawables

Drawables are simply `sketch` objects that might appear in the drawing. They include dots, polylines, curves, polygons, and more complex objects that are built up from simpler ones in various ways. Finally, *special* objects are those composed of LaTeX or `PSTricks` code, perhaps including coordinates and angles computed by `sketch`.

3.2.1 Dots

Syntax:

```
dots[options] point_list
```

This command is the three-dimensional equivalent of the `PSTricks` command `\psdots`.

3.2.2 Lines

Syntax:

```
line[options] point_list
```

This command is the three-dimensional equivalent of the `PSTricks` command `\psline`.

3.2.3 Curves

Syntax:

```
curve[options] point_list
```

This command is the three-dimensional equivalent of the `PSTricks` command `\pscurve`. **It is not implemented in the current version of** `sketch`.

3.2.4 Polygons

Syntax:

```
polygon[options] point_list
```

This command is the three-dimensional equivalent of the `PSTricks` command `\pspolygon`. The `sketch` hidden surface algorithm assumes that polygons are convex and planar. In practice, drawings may well turn out correctly even if these assumptions are violated.

3.2.5 Specials

Syntax:

```
special $raw_text$[lay=lay_value] point_list
```

Here `$` can be any character and is used to delimit the start and end of *raw_text*. The command embeds *raw_text* in the `sketch` output after performing substitutions as follows.

- `#i` where *i* is a positive integer is replaced by the *i*'th point in *point_list*.
- `#{i}` is also replaced as above.
- `#i-j` where *i* and *j* are positive integers is replaced by a string `{angle}` where *angle* is the polar angle of a vector from the *i*'th point in *point_list* to the *j*'th.
- `#{i-j}` is also replaced as above.
- `##` is replaced with `#`.

The forms with braces '`{ }`' are useful when the argument is immediately followed by a digit that is part of the TEX code.

The only useful option of `special` is `lay`. See Section 3.1.5.6 [Internal options], page 16.

3.2.6 Sweeps

Syntax:

```
sweep { n, T_1, T_2, ..., T_r }[options] swept_object
sweep { n<>, T_1, T_2, ..., T_r }[options] swept_object
```

The sweep connects n (or perhaps $n+1$) copies of *swept_object* in order to create a new object of higher dimension. The *T_i* (for i between 1 and r) are transforms. The k'th copy of *swept_object* is produced by applying the following transform to the original.

$T_1{}^k$ `then` $T_2{}^k$ `then ... then` $T_r{}^k$

Here T^k means "transform T applied k times." The original object is the zero'th copy, with $k = 0$ and effectively no transform applied ($T^0 = I$, the identity transform).

The method of connecting the copies depends on the type of *swept_object* and on whether the closure tag '`<>`' is present or not.

An example of a sweep where $r = 2$ is the Mobius figure at Section 2.7.7 [More to learn], page 10.

3.2.6.1 Swept points

If *swept_object* is a point list and there is no closure tag, then `sweep` connects $n+1$ successive copies of each point (including the original) with straight line segments to form a polyline. If there are m points in the original point list, then m polylines with n segments each are formed by the sweep. In this manner, `sweep` forms a set of one-dimensional objects (polylines) from zero-dimensional ones (points).

When there *is* a closure tag, `sweep` connects n successive copies of each point (including the original) with straight line segments and finally connects the last copy back to the original to form a polygon with n sides. If there are m points in the original point list, then m polygons with n sides each are formed by the sweep. In this manner, `sweep` forms a set of two-dimensional objects (polygons) from zero-dimensional ones (points).

Options of the `sweep` are copied directly to the resulting polyline(s).

3.2.6.2 Swept lines

If *swept_object* is a polyline and there is no closure tag, then `sweep` connects $n+1$ successive copies of the polyline (including the original) with four-sided polygons, each pair of copies giving rise to a "polygon strip." If there are m points in the original polyline, then $(m-1)n$

polygons are formed by the sweep. We call these *body polygons*. In this manner, `sweep` forms a two-dimensional surface from from a one-dimensional polyline.

The order of vertices produced by `sweep` is important. If a polygon's vertices do not appear in counter-clockwise order in the final image, the polygon will be culled (unless `cull=false` is set). If the points in the k'th copy of the polyline are P_1, P_2, ..., P_m, and the points in the next copy, the $(k+1)$st, are P'_1, P'_2, ..., P'_m, then the vertex order of the generated polygons is

Body polygon 1: $P_2\ P_1\ P'_1\ P'_2$
Body polygon 2: $P_3\ P_2\ P'_2\ P'_3$
...
Body polygon $m-1$: $P_m\ P_{m-1}\ P'_{m-1}\ P'_m$

Options of unclosed line sweeps are copied to each output polygon. Options of the swept line are ignored.

When there *is* a closure tag, then `sweep` connects n successive copies of the polyline (including the original) with four-sided body polygons just as the case with no closure tag. It then connects the last copy back to the original to form a ribbon-shaped surface that closes on itself with two holes remaining.

Finally, the sweep adds two more polygons to seal the holes and form a closed surface that, depending on the sweep transforms, may represent the boundary of a solid. In this manner, `sweep` forms the boundary of a three-dimensional object from a one-dimensional polyline. We call these hole-filling polygons *ends*.

The order of vertices of end polygons is important for correct culling as described above. If P_1^1, P_1^2, ..., P_1^n are the n copies of the first polyline point and P_m^1, P_m^2, ... ,P_m^n are the n copies of the last polyline point, then the end polygon vertex order is

End polygon 1: P_1^n, P_1^{n-1}, ... ,P_1^1
End polygon 2: P_m^1, P_m^2, ... ,P_m^n

If there are no options on the swept line, then the '`sweep`' options are copied to each output polygon. If the swept line does have options, these are copied to corresponding body polygons; the sweep options are copied to the end polygons. In this manner, body and ends may be drawn with different characteristics such as `fillcolor`.

3.2.6.3 Swept polygons

If *swept_object* is a polygon, the `sweep` connects $n+1$ successive copies of the closed polyline border of the polygon to form body polygons exactly as though the border were a swept polyline as described in Section 3.2.6.2 [Swept lines], page 18. If there are m points in the original polygon, then mn body polygons are formed by this sweep. The body polygons form an *extrusion* of the boundary of the original polygon with two holes at the open ends.

Finally, the sweep adds two copies of the original polygon to cover the holes. We call these hole-filling polygons *ends*. In this manner, `sweep` forms the boundary of a three-dimensional object from a two-dimensional polygon.

The order of vertices of end polygons is important for correct culling as described above. An exact copy of the original polygon with vertex order intact forms the first end polygon. The other end polygon results from transforming and the reversing the order of vertices in the original. The transform places the original polygon at the uncovered hole; it is

$T_1{}^n$ `then` $T_2{}^n$ `then ... then` $T_r{}^n$`.`

If there are no options on the swept polygon, then the '`sweep`' options are copied to each output polygon. If the swept polygon does have options, these are copied to the ends; the sweep options are copied to the body polygons. In this manner, body and ends may be drawn with different characteristics such as `fillcolor`.

3.2.6.4 Swept blocks

The swept object *swept_object* may also be any collection of polylines and polygons. This may be a block composed of `line` and/or `polygon` commands in braces '`{ }`', or it may be the result of a `repeat`, another `sweep`, etc. The sweep acts independently on each object in the block exactly as if it were a single swept object described above in Section 3.2.6.2 [Swept lines], page 18 and Section 3.2.6.3 [Swept polygons], page 19.

3.2.6.5 Sweep face splitting

Before sending each four-sided body polygon of a `sweep` to the output, `sketch` tests to see if it is roughly planar. Since planarity is necessary for proper functioning of the hidden surface algorithm, "warped" polygons are automatically split into two triangles.

Hole-filling polygons produced by closure-tagged line sweeps are not split. Nor are original polygons in polygon sweeps. It is the user's responsibility to ensure these are planar.

3.2.7 Blocks

Any sequence of drawables may be grouped in a *block* merely by enclosing them in braces '`{ }`'. A block is itself drawable. A key use of blocks is to extend the effect of a single `def`, Section 3.3 [Definitions], page 21, `put` Section 3.2.9 [Puts], page 20, `sweep` Section 3.2.6 [Sweeps], page 18, or `repeat` Section 3.2.8 [Repeats], page 20 to include several objects rather than one.

Definitions (See Section 3.3 [Definitions], page 21.) inside a block have *lexical scope* extending from the place of definition to the end of the block.

3.2.8 Repeats

Syntax:

```
    repeat { n, T_1, T_2, ..., T_r } repeated_object
```

The repeat makes n transformed copies of *repeated_object* (including the original). The *T_i* are transforms. The k'th copy of the *repeated_object* (for $k = 0, 1, ..., n-1$) is produced in the same manner as for `sweeps` described in Section 3.2.6 [Sweeps], page 18. This is repeated here (no pun intended) for convenience. To make the k'th copy, the following transform is applied to the original object.

$T_1{}^k$ `then` $T_2{}^k$ `then ... then` $T_r{}^k$

Here T^k means "transform T applied k times."

3.2.9 Puts

Syntax:

```
    put { T } put_object
```

Put merely applies transform T to the drawable *put_object*.

3.3 Definitions

Definitions give names to `sketch` objects. Definitions alone are benign. A `sketch` input file consisting entirely of definitions will generate no drawing. Only when definitions are *referenced* do they potentially lead to ink on the drawing.

The intent of definitions is to make `sketch` code more concise and readable. There is no input file employing definitions that could not be re-written without them.

Definable objects include any result of an affine arithmetic expression (scalar, point, vector, or transform), any drawable object (dots, line, curve, polygon, block, sweep, put, repeat, or special), and option strings. In addition, *tag definitions*, which have no associated object at all, allow the meaning of other definitions to be selected from a set of alternatives. Since tags may be defined (and undefined) in the command line of `sketch`, they can be an aid in the script-driven preparation of documents.

3.3.1 Forms of definitions

Definitions have three possible forms, *simple*, *with alternatives*, and *tag* as shown here in order.

Syntax:

```
def id object  % simple def
def id <tag_1> object_1  % def with alternatives
       <tag_2> object_2
       ...
       <> default_object
def id <>  % tag def
```

The simple definition merely associates *object* with the identifier *id*.

The definition with alternatives associates *object_i* with *id*, where *tag_i* is the first defined tag in the list of alternative tag references. If no tag in the list is defined, then *default_object* is associated with identifier *id*.

The final form defines *id* as a tag. Another way to define a tag is with the '`-D`' command line option. See Chapter 5 [Command line], page 36.

3.3.2 Forms of references

References to defined names are enclosed in bracketing delimiters. The delimiter characters imply the type of the associated value as shown in the table below. A type error is raised if the type of a reference does not match the type of the defined value. The intent of this mechanism is, again, to make `sketch` input files more readable.

Type	**Reference**
scalar	`id`
point	`(id)`
vector	`[id]`
transform	`[[id]]`
drawable	`{id}`
options	`[id]` or `[id1,...,idN]`
tag	`<id>`

Note that square brackets '`[ ]`' are used both for vector and for options references. Details of `sketch` syntax make it impossible for these two reference types to be confused. The special multiple reference `[`*`id1`*`,`*`id2`*`,...,`*`idN`*`]` acts as if the respective lists of options were concatenated.

3.4 Global environment

An optional global environment block provides a few ways to affect the entire scene. The block must appear as the last text in the `sketch` input file. It may include definitions, but note that previous definitions at the top level (not nested inside blocks) are also available.

Syntax:

```
global { environment_settings }
```

The contents of *environment_settings* are discussed in the sections that follow.

3.4.1 Global options

Syntax:

```
set [ options ]
```

The contents of *options*, except for `sketch` internal options, are copied as-is to a `\psset` that appears before anything else in the output file. This is a good place to set `unit`, a default `linewidth`, etc.

Internal options work on all objects where they make sense. This includes `cull` and `split` (but not `lay`). See Section 3.1.5.6 [Internal options], page 16.

3.4.2 Camera

Syntax:

```
camera transform_expression
```

The *transform_expression* is applied after all other transformations of the scene. This is currently only useful for transforming the bounding box. See Section 3.4.3 [Picture box], page 22. It will play a role in any future implementation of clipping.

3.4.3 Picture box

Syntax:

```
picturebox[baseline]
picturebox[baseline] (p1)(p2)
```

The first form of `picturebox` causes a scalar *baseline* fraction to be emitted in the `pspicture` environment of the output. See `PSTricks` documentation for `pspicture`.

In the second form, the *baseline* fraction is optional, and the two points that follow define the diagonal of a three-dimensional bounding box for the completed scene. The parallel projection of the bounding box determines the corners of the drawing's `pspicture*` environment, which is used in place of `pspicture`. This causes PostScript to clip the final drawing to the bounding box in 2d. If there is a `camera` specified, the camera tranformation is applied to the bounding box, and the `pspicture` is set just large enough to include the transformed box.

When no bounding box is given, `sketch` computes one automatically.

3.4.4 Frame

Syntax:

```
frame [options]
```

Causes a `\psframebox` to surround the `pspicture` environment in the output. If *options* are present, they are copied as-is. Normally one would want to set `linewidth`, `linestyle`, `linecolor`, etc. If omitted, then `framesep=0pt` is added so that the frame tightly hugs the `pspicture`.

3.4.5 Language

```
language tikz
language tikz, context
language pstricks
language pstricks, latex
```

Sets the output language generated by `sketch`. The set of options understood by sketch also changes. For example, the `PSTricks` option `linewidth` will not be properly handled if `language` is set to `tikz`. Similarly, the `TikZ` option `line style` (note the space) will not be properly handled if `language` is set to `pstricks`. If no language is specified, the default `pstricks` is used.

An optional comma followed by `latex` or `context` specifies the macro package that the output should assume. This affects the `picture` environment commands emitted and the document template used with the '`-T`' option. See Chapter 5 [Command line], page 36. Note that at the time this manual was generated, `PSTricks` was not supported by LaTeX or by `ConTeXt`.

4 Building a drawing

Successful drawings with `sketch` and with any scene description language require that the user develop an accurate mental picture of her code and its meaning. This image is best built in small pieces. Therefore, `sketch` inputs are best created in small increments with frequent pauses to compile and view the results. Careful comments in the input often help as a scene grows in complexity.

4.1 Overview

As an overview, let's develop a diagram that shows how a perspective projection transform works. We'll start with the traditional reference object used in computer graphics textbooks, a house-shaped prism. Begin by defining the points of the house. Rather than defining the faces of the house as polygons and transforming those, we are going to transform the points themselves with `sketch` arithmetic so that we have names for the transformed points later.

```
% right side (outside to right)
def R1 (1,1,1) def R2 (1,-1,1) def R3 (1,-1,-1) def R4 (1,1,-1)
def R5 (1,1.5,0)

% left side (outside to right--backward)
def W  [2,0,0]
def L1 (R1)-[W] def L2 (R2)-[W] def L3 (R3)-[W] def L4 (R4)-[W]
def L5 (R5)-[W]
```

To add a door to the house, we use a polygon slightly in front of the foremost face of the house.

```
% door
def e .01
def D1 (0,-1,1+e) def D2 (.5,-1,1+e) def D3 (.5,0,1+e) def D4 (0,0,1+e)
```

Now let's create a new set of points that are a to-be-determined transform of the originals.

```
def hp scale(1) % house positioner
def pR1 [[hp]]*(R1) def pR2 [[hp]]*(R2) def pR3 [[hp]]*(R3)
def pR4 [[hp]]*(R4) def pR5 [[hp]]*(R5)
def pL1 [[hp]]*(L1) def pL2 [[hp]]*(L2) def pL3 [[hp]]*(L3)
def pL4 [[hp]]*(L4) def pL5 [[hp]]*(L5)
def pD1 [[hp]]*(D1) def pD2 [[hp]]*(D2) def pD3 [[hp]]*(D3)
def pD4 [[hp]]*(D4)
```

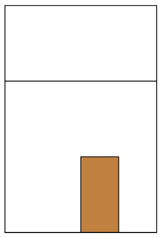
Note the use of a *transform definition* and *transform references*. Now define the seven polygonal faces of the house and the door using the transformed points as vertices. Be careful with vertex order!

```
def rgt polygon (pR1)(pR2)(pR3)(pR4)(pR5)
def lft polygon (pL5)(pL4)(pL3)(pL2)(pL1)
def frt polygon (pR2)(pR1)(pL1)(pL2)
def bck polygon (pR4)(pR3)(pL3)(pL4)
def tfr polygon (pR1)(pR5)(pL5)(pL1)
def tbk polygon (pR5)(pR4)(pL4)(pL5)
def bot polygon (pR2)(pL2)(pL3)(pR3)
def door polygon[fillcolor=brown] (pD1)(pD2)(pD3)(pD4)
def house { {rgt}{lft}{frt}{bck}{tfr}{tbk}{bot}{door} }
```

Time for a sanity check. Add the line

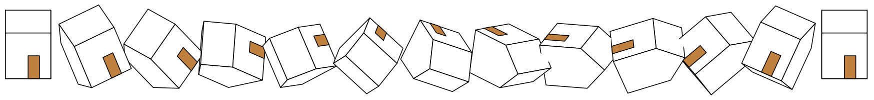
```
{house}
```

and this is what we get.



This is correct, but does not reveal very much. Common errors are misplaced vertices and polygons missing entirely due to incorrect vertex order. To rule these out, let's inspect all sides of the house. This is not hard. Merely replace the reference `{house}` with a `repeat`. See Section 3.2.8 [Repeats], page 20.

```
repeat { 13, rotate(30, [1,2,3]), translate([3,0,0]) } {house}
```



Again things look correct. Note that the hidden surface algorithm handles intersecting polygons correctly where some copies of the house overlap.

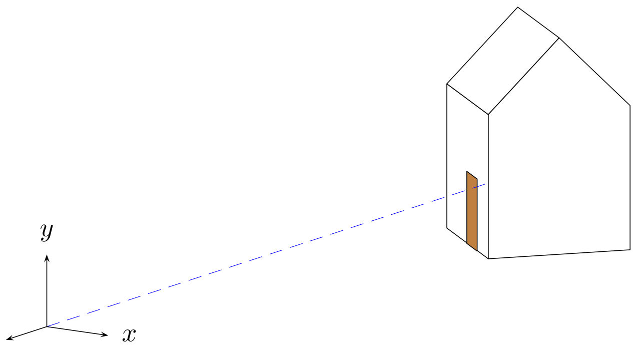
Let's lay out the geometry of perspective projection of the house onto a plane with rays passing through the origin. Begin by positioning the house twelve units back on the negative z-axis and adding a set of coordinate axes. To move the house we need only change the "house positioning" transform defined earlier.

```
def hp rotate(-40, [0,1,0]) then translate([0,0,-12])
def axes {
  def sz 1
  line [arrows=<->] (sz,0,0)(O)(0,sz,0)
  line [arrows=->]  (O)(0,0,sz)
  line [linewidth=.2pt,linecolor=blue,linestyle=dashed] (O)(0,0,-10)
  special |\uput[r]#1{$x$}\uput[u]#2{$y$}\uput[l]#3{$z$}|
    (sz,0,0)(0,sz,0)(0,0,sz)
}
```

Time for another test. Let's build a real view transform, creating a *virtual camera* to look at the scene we are constructing. Replace the `repeat` with

```
def eye (10,4,10)
def look_at (0,0,-5)
put { view((eye), (look_at)) } { {house}{axes} }
```

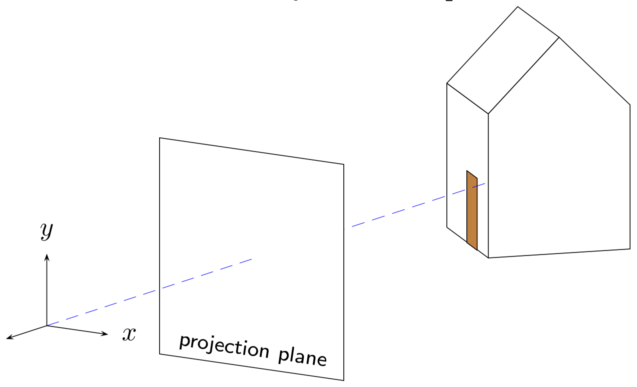
The *view transform* repositions the scene so that the point `eye` is at the origin and the direction from `eye` to `look_at` is the negative z-axis. This requires a rotation and a translation that are all packed into the constructor `view`.



This is starting to look good! Add the projection plane half way between the origin and the house at $z = -5$. We'll try the angle argument feature of `special` to position a label.

```
def p 5 % projection distance (plane at z=-p)
def projection_plane {
  def sz 1.5
  polygon (-sz,-sz,-p)(sz,-sz,-p)(sz,sz,-p)(-sz,sz,-p)
  special |\rput[b]#1-2#3{\footnotesize\sf projection plane}|
    (-sz,-sz,-p)(sz,-sz,-p)(0,-sz+.1,-p)
}
```

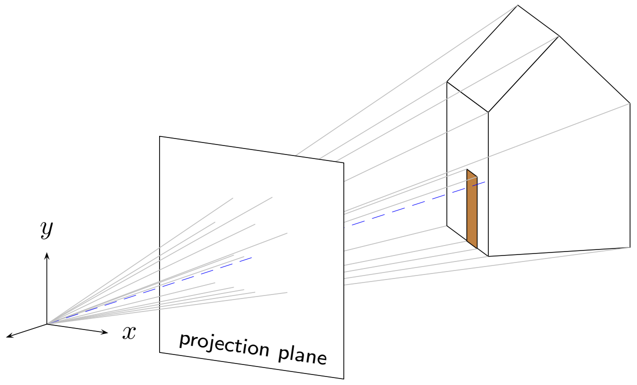
Add `{projection_plane}` to the list of objects in the `put` above.



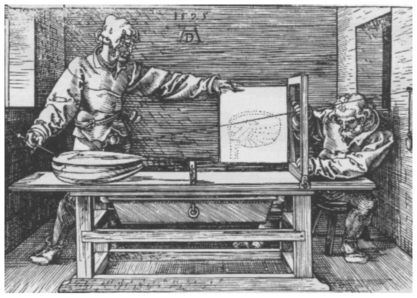
The way we constructed the points of the house now makes it easy to draw rays of projection. We'll cast one ray from every visible vertex of the house and define options so the appearance of all rays can be changed at the same time.

```
def projection_rays {
  def rayopt [linewidth=.3pt,linecolor=lightgray]
  line [rayopt](O)(pR1) line [rayopt](O)(pR2) line[rayopt](O)(pR3)
  line [rayopt](O)(pR4) line [rayopt](O)(pR5)
  line [rayopt](O)(pL1) line [rayopt](O)(pL2) line[rayopt](O)(pL5)
  line [rayopt](O)(pD1) line [rayopt](O)(pD2)
  line [rayopt](O)(pD3) line [rayopt](O)(pD4)
}
```

The result is shown here.



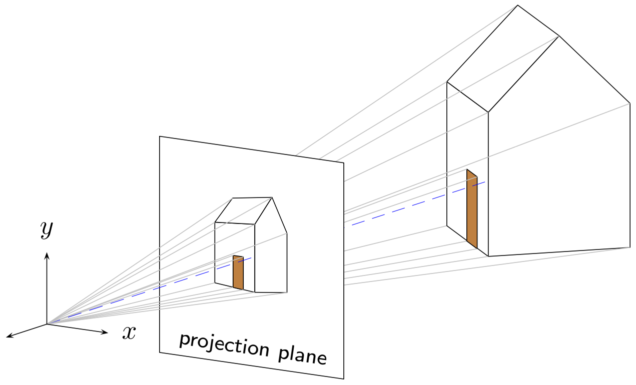
The rays pierce the projection plane at the corresponding points on the perspective image we are trying to draw. Albrecht Dürer and his Renaissance contemporaries had the same idea in the early 1500's.



All that's left is to find a way to connect the points of the house on the projection plane. We could pull out a good computer graphics text, find the necessary matrix, and enter it ourselves as a transform literal. See Section 3.1.3.3 [Transform literals], page 12. That work is already done, however. We can use the `project(p)` constructor.

There are still some details that require care. Projection will flatten whatever is transformed onto the plane $z = -p$. Therefore any part of the house could disappear behind the projection plane (the hidden surface algorithm orders objects at the same depth arbitrarily). The door may also disappear behind the front of the house. To make sure everything remains visible, we'll place the house a tiny bit in front of the projection plane and a second copy of the door in front of the house.

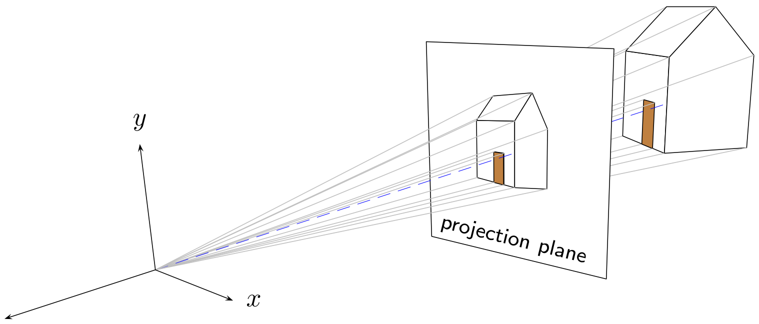
```
def projection {
  % e is a small number defined above
  put { project(p) then translate([0,0,1*e]) } {house}
  put { project(p) then translate([0,0,2*e]) } {door}
}
```



If you have studied and understand all this, you are well on the way to success with `sketch`. Not shown are the 20 or so iterations that were required to find a reasonable viewing angle and house position, etc. Nonetheless, this drawing was completed in about an hour. While a GUI tool may have been a little faster, it is unlikely that a new drawing, itself a perspective projection of the scene, could be generated with two more minutes' work! Just change the view transform to

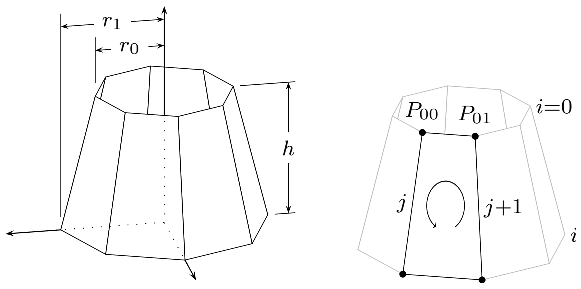
```
put { view((eye), (look_at)) then perspective(9) } { ...
```

and produce this.



4.2 A technical drawing

Let's look at a drawing that represents the kind of problem `sketch` was meant to solve—a pair of textbook figures regarding a polygonal approximation of a truncated cone. Here are the pictures we will produce.



The cone shape is just a swept line with no closure tag and culling turned off. Begin by setting up some useful constants.

```
def O (0,0,0) def I [1,0,0] def J [0,1,0] def K [0,0,1]
def p0 (1,2) def p1 (1.5,0) def N 8
def seg_rot rotate(360 / N, [J])
```

The points p0 and p1 are the end points of the line to be swept. The definition seg_rot is the sweep transformation. With these, the cone itself is simple.

```
sweep[cull=false] { N, [[seg_rot]] } line(p0)(p1)
```

The axes are next and include an interesing trick that shows the hidden parts as dotted lines. The secret is draw the axes twice—solid lines with the normal hidden surface algorithm in effect, and then dotted with the option lay=over so that no polygons can hide them.

```
def ax (dx,0,0) % tips of the axes
def ay (0,dy,0)
def az (0,0,dz)
line[arrows=<->,linewidth=.4pt](ax)(O)(ay)
line[arrows=->,linewidth=.4pt](O)(az)
% repeat dotted as an overlay to hint at the hidden lines
line[lay=over,linestyle=dotted,linewidth=.4pt](ax)(O)(ay)
line[lay=over,linestyle=dotted,linewidth=.4pt](O)(az)
special|\footnotesize
         \uput[d]#1{$x$}\uput[u]#2{$y$}\uput[l]#3{$z$}|
  (ax)(ay)(az)
```

The labels are applied with PSTricks special objects as usual.

For the height dimension mark, the power of affine arithetic is very helpful.

```
def hdim_ref unit((p1) - (O)) then [[seg_rot]]^2
def c0 (p0) then scale([J])
def h00 (c0) + 1.1 * [hdim_ref]
def h01 (c0) + 1.9 * [hdim_ref]
def h02 (c0) + 1.8 * [hdim_ref]
line(h00)(h01)
def h10 (O) + 1.6 * [hdim_ref]
def h11 (O) + 1.9 * [hdim_ref]
def h12 (O) + 1.8 * [hdim_ref]
line(h10)(h11)
line[arrows=<->](h02)(h12)
def hm2 ((h02) - (O) + (h12) - (O)) / 2 + (O)
special|\footnotesize\rput*#1{$h$}|(hm2)
```

The general idea employed here is to compute a unit "reference vector" parallel to the xz-plane in the desired direction of the dimension from the origin. The transformation [[seg_rot]]^2 rotates two segments about the y-axis. When applied to (p1) - (O), the resulting vector points to the right as shown. In this manner, we can pick any vertex as the location of the height dimension lines by varying the exponent of [[seg_rot]]. This is only one of many possible strategies.

The computation of hm2 is a useful idiom for finding the centroid of a set of points.

The two radius marks are done similarly, so we present the code without comment.

```
% radius measurement marks
def gap [0,.2,0]  % used to create small vertical gaps

% first r1
```

```
def up1 [0,3.1,0] % tick rises above dimension a little
def r1 ((p1) then [[seg_rot]]^-2) + [up1]
def r1c (r1) then scale([J])
def r1t (r1) + [gap]
def r1b ((r1t) then scale([1,0,1])) + [gap]
line[arrows=<->](r1c)(r1)  % dimension line
line(r1b)(r1t)             % tick
def r1m ((r1) - (O) + (r1c) - (O)) / 2 + (O) % label position
special |\footnotesize\rput*#1{$r_1$}|(r1m)  % label

% same drill for r0, but must project down first
def up0 [0,2.7,0]
def r0 ((p0) then scale([1,0,1]) then [[seg_rot]]^-2) + [up0]
def r0c (r0) then scale([J])
def r0t (r0) + [gap]
def r0b ((p0) then [[seg_rot]]^-2) + [gap]
line[arrows=<->](r0c)(r0)
line(r0b)(r0t)
def r0m ((r0) - (O) + (r0c) - (O)) / 2 + (O)
special |\footnotesize\rput*#1{$r_0$}|(r0m)
```

The second drawing uses the same techniques. Only the method for drawing the elliptical arc is new. Here is the code.

```
def mid ((p00)-(O)+(p10)-(O)+(p11)-(O)+(p01)-(O))/4+(O)
special|\rput#1{\pscustom{
  \scale{1 1.3}
  \psarc[arrowlength=.5]{->}{.25}{-60}{240}}}|
  [lay=over](mid)
```

We could have swept a point to make the arc with `sketch`, but using a `PSTricks` custom graphic was simpler. Again we computed the centroid of the quadrilateral by averaging points. Note that scaling in Postscript distorts the arrowhead, but in this case the distortion actually looks better in the projection of the slanted face. A `sketch` arrowhead would not have been distorted.

The complete code for this example, which draws either figure depending on the definition of the tag `<labeled>`, is included in the `sketch` distribution in the file '`truncatedcone.sk`'.

4.3 A hierarchical model

While `sketch` was never meant to be a geometric modeling language, it comes fairly close. The following example puts all we have seen to work in a very simple model of the human hand. Start by sweeping a line to make a truncated cone, which will be copied over and over again to make the segments of fingers.

```
def O (0,0,0) % origin
def I [1,0,0] def J [0,1,0] def K [0,0,1] % canonical unit vectors
def segment {
  def n_faces 8
  sweep { n_faces<>, rotate(360 / n_faces, [J]) }
```

```
    line(proximal_rad, 0)(distal_rad, distal_len)
}
```

In hand anatomy, *distal* is "at the tip" and *proximal* is "in the area of the palm." We have omitted all the scalar constants. You can find them in 'hand.sk', which is provided in the sketch distribution.

We also need a prototypical sphere to use for the joints themselves.

```
def joint_sphere {
  def n_joint_faces 8
  sweep [fillcolor=red] { n_joint_faces, rotate(360 / n_joint_faces, [J]) }
    sweep { n_joint_faces, rotate(180 / n_joint_faces) }
      (0, -joint_rad)
}
```

We'll now design the index finger (number 1 in our notational convention; finger 0 is the thumb). The distal rotation for the finger applies only to the tip, so we define the following.

```
def distal_1 {
  put { translate(joint_gap * joint_rad * [J])
        then rotate(distal_1_rot, [I])
        then translate((distal_len + joint_gap * joint_rad) * [J]) }
    {segment}
  put { rotate(distal_1_rot / 2, [I])
        then translate((distal_len + joint_gap * joint_rad) * [J]) }
    {joint_sphere}
  put { scale( [J] + proximal_distal_ratio * ([I]+[K]) ) }
    {segment}
}
```

The identifiers here are for size and location constants. The exception is distal_rot_1. This rotation parameter models the flexing of the finger tip. The first put makes a copy of the finger segment that is translated upward just far enough to make room for the spherical joint. Then it applies the distal rotation. Finally it translates the whole assembly upward again to make room for the middle phlanges (the next bone toward the palm). The second put positions the sphere. There is a rotation to place the grid on the sphere surface at an nice angle, then a translation to the base of the distal phlanges, which is also center of its rotation. Finally, the last put positions the middle segment itself.

The middle joint is the next one down, with rotation angle middle_rot_1. When this angle changes, we need all the objects in distal_1 to rotate as a unit. This is the reasoning behind the next definition.

```
def finger_1 {
  put { translate(joint_gap * joint_rad * [J])
        then rotate(middle_1_rot, [I])
        then translate((middle_ratio * distal_len +
                        joint_gap * joint_rad) * [J]) }
    {distal_1}
  put { scale(proximal_distal_ratio)
        then rotate(middle_1_rot / 2, [I])
        then translate((middle_ratio * distal_len +
                        joint_gap * joint_rad) * [J]) }
    {joint_sphere}
  put { scale( middle_ratio * [J] +
               proximal_distal_ratio^2 * ([I]+[K]) ) }
    {segment}
}
```

This looks very similar to the previous definition, and it is. The important difference is that rather than positioning and rotating a single segment, we position and rotate the entire "assembly" defined as `distal_1`. The rest is just arithmetic to compute sizes and positions that look nice. The last `put` places an appropriately shaped segment that is the *proximal phlanges*, the bone that joins the palm of the hand. This completes the finger itself.

All the other fingers are described identically to this one. We account for the fact that real fingers are different sizes in the next step, which is to build the entire hand.

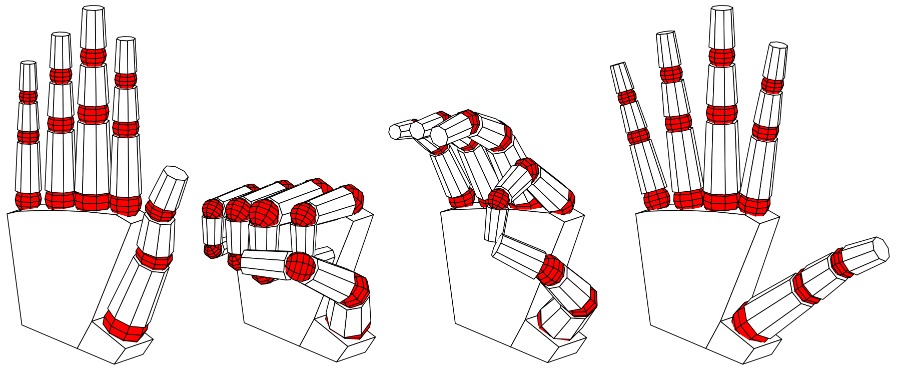
The `hand` definition that follows includes a section for each finger. We'll continue with finger 1 and omit all the others. (Of note is that the thumb needs slightly special treatment—an extra rotation to account for its opposing angle. This is clear in the full source code.) Not surprisingly, the `hand` definition looks very much like the previous two. It should be no surprise that when the rotation parameter `meta_1_rot` changes, the entire finger rotates! There is an additional rotation that allows the fingers to spread laterally. We say these joints of the proximal phlanges have two *degrees of freedom*. The joints higher on the finger have only one. Finally, each finger is scaled by a factor to lend it proportion.

```
  def hand {
    % finger 1 [all other fingers omitted]
    def scale_1 .85
    put { scale(scale_1)
          then translate((joint_gap * joint_rad) * [J])
          then rotate(meta_1_rot, [I])
          then rotate(-spread_rot, [K])
          then translate((proximal_1_loc) - (O)) }
      {finger_1}
    put { scale(scale_1 * proximal_distal_ratio^2)
          then rotate(meta_1_rot / 2, [I])
          then rotate(-spread_rot, [K])
          then translate((proximal_1_loc) - (O)) }
      {joint_sphere}

    % palm
    sweep { 1, rotate(6, (0,15,0), [I]) }
      put { rotate(-3, (0,15,0), [I]) } {
        polygon(proximal_1_loc)(proximal_2_loc)
               (proximal_3_loc)(proximal_4_loc)
               (h5)(h6)(h6a)(h9)(h10)
        polygon(h6a)(h7)(h8)(h9)
   }  }
```

The last section of the definition creates the polytope for the palm of the hand by `sweeping` a 10-sided polygon through a very short arc (9 degrees). This provides a wedge-shaped profile when viewed from the side. The thick end of the wedge is the wrist. Because the polygon is concave, it is split into into two convex shapes with nine and four vertices.

We can now have fun positioning the hand by adjusting the various rotation angles. The complete source includes definitions with alternatives that include the following views and more.



4.4 Caveats

`Sketch` is a fairly powerful tool for drawing, but, just as with TEX, the power to create beautiful results comes along with the power to make mistakes. The following are some points where care is necessary and where the current version of `sketch` is limited or has known bugs.

4.4.1 Limits on `sketch` error detection

`Sketch` catches many kinds of errors, but not all. For example, options that sketch does not recognize, even incorrect ones, are quietly copied to `PSTricks` commands in the output. It is also unfortunately easy to produce `sketch` inputs that lead to no picture at all (improper vertex ordering causes everything to be culled), to pictures that are too big or too small for `PSTricks` to draw (due to limits of TEX math), and pictures that look nothing like what was intended. A picture with one of these problems can be difficult to "debug." We offer the following suggestions.

- Follow the suggested incremental development method described in Section 4.1 [Overview], page 24.
- Always maintain one or two back-versions of a drawing so that it is easy to fall back to a known-good increment.
- When using `perspective`, ensure all finally transformed objects satisfy $z < 0$ and, in fact, do not come very close to the origin at all.
- Temporarily use `cull=false` to see where vertex ordering problems lie.
- Use temporary changes of color of one or more objects to ensure that your understanding of the scene geometry is correct.
- If `PSTricks` complains about something, inspect the output directly for clues.

4.4.2 Clipping

The current version of `sketch` has no clipping operations. The entire scene is always drawn. This means that when a perspective transform is employed, it is the user's responsibility to make sure the entire scene remains in front of the viewer, the region $z < 0$.

4.4.3 Hidden surface removal and polygon splitting

`Sketch` uses the *depth sort algorithm* for hidden surface removal. This is a very old technique due to Newell.[1] It is generally regarded as too slow for real time graphics, but it is ideal for our purpose where speed is not very important.[2]

The depth sort algorithm merely sorts objects on a key of increasing z-coordinate, equivalent to decreasing depth. Objects are then drawn in the sorted sequence so that those at the rear of the scene are overwritten by those closer to the viewer. Since this is also how oil painters practice their art, depth sort is sometimes called "the painter's algorithm."

In some cases it is impossible to strictly order polygons according to depth. Moreover, even if a correct depth ordering exists, the computation needed to find it may be too complex and slow. In these cases, `sketch` splits one or more polygons into pieces. The expectation is that the new, smaller polygons will be simpler to order. `Sketch` uses a BSP (binary space partition) to handle the splitting operation.

4.4.3.1 Statistics

For the curious, `sketch` writes one line of depth sort statistics. Here is an example for a large collection of triangles.

```
remark, node=34824 probe=581.9 swap=5 split=2 (in=4 out=6) ols=24851/0
```

It means that $34,824$ objects were depth sorted after culling. For each, an average of 581.9 others had to be checked to ensure that the initial, approximate ordering was correct. Among all these checks, only 5 resulted in swaps to reorder the initial sort. In two cases, a correct ordering could not be determined, so binary space partitions were constructed for splitting. A total of 4 objects (triangles in this case) were inserted in the partitions, and 6 polygons were produced. Finally, $24,851$ "last resort" polygon overlap checks were performed after simpler, faster checks failed to yield conclusive results. The final /0 is for line-polygon overlap checks. For comparison, the statistics for the last figure in Section 4.1 [Overview], page 24 follow.

```
remark, node=27 probe=14.6 swap=36 split=15 (in=30 out=45) ols=0/69
```

Note that there was proportionally much more swapping and splitting activity in this highly connected scene.

4.4.3.2 Bugs and anomalies

Polygon and line splitting can both cause anomalies in the output. `PSTricks` dash patterns, specified with `linestyle=dashed`, can be disrupted by splitting. This occurs when the

[1] Newell, M.E., R.G. Newell, and T.L. Sancha, A solution to the hidden surface problem. *Proceedings of the ACM annual conference - Volume 1*, page 443–450, ACM Press, 1972.

[2] We have run `sketch` on the famous Stanford Bunny, which consists of nearly $70,000$ triangles. Run time was about 6 seconds. Most of this was spent writing the output file rather than in the hidden surface algorithm. LaTeX took much longer to process the resulting `PSTricks` code. The obvious conclusion is that the speed of the depth sort algorithm is not a worry.

depth sort gives up too early and splits a line where it is not really necessary. A workaround is to use gray or finely dotted lines instead. If your drawing is small, you can also edit the `sketch` output by hand to merge the pieces of the offending line.

Another anomaly is tiny (or in degenerate cases not-so-tiny) notches in the lines that border split polygons. These derive from the way each polygon is painted: first, all pixels within the boundary are *filled* with color (perhaps white), then the same boundary is *stroked* (a Postscript term) with a line. The result is that half the line lies inside the boundary and half outside, while the Painter's algorithm assumes the polygon lies entirely within its boundary. The notches are due to one polygon fill operation overwriting the already-drawn inside of the border of another polygon.[3] One workaround is to make border lines very thin. In fact `linewidth=0pt` is guaranteed to eliminate this problem, though this results in the thinnest line your output device can draw, which is usually too thin. You might get lucky by merely reordering things in the input file, which is likely to move the splits to different places. The only sure-fire solution is pretty terrible: custom fit `special` overlay lines (with `\psline`) to cover the notches.

Polygon splitting also breaks `PSTricks` hatch patterns. The only known workaround is to substitute a solid fill for the hatch.

[3] I know how to fix this problem, but I don't like my solution, and I'm interested in yours.

5 Command line

Synopsis:

```
     sketch [-h][-V x.y][-v][-b][-d][t doctmp][-T[u|e][p[P|T][L|C]]][-o output.tex]
       [-D tag ...] input1.sk [-U tag ...] input2.sk ...
```

Description Processes the `sketch` input files in order to produce `PSTricks` output code suitable for inclusion in a TEX or LATEX document.

Options:

`-h` Print a short catalog of options.

`-V` Set the `PSTricks` version assumed for output purposes to `x.y`, for example 1.19. Usually needed only if your `PSTricks` is old compared to your `sketch`. Use `-v` to see what `sketch` assumes by default.

`-v` Print version information to standard output, including the version of `PSTricks` assumed for output (can be changed with `-V` above).

`-b` Use a BSP (See Section 4.4.3 [Hidden surface removal], page 34.) for *all* hidden surface removal rather than the default, which is the depth sort algorithm with BSPs used only for cycle resolution. This may produce correct output in certain degenerate cases where the depth sort cannot, but it also leads to many gratuitous splits, hence more anomalies Section 4.4.3.2 [Bugs and anomalies], page 34 and big output files.

`-d` Run `sketch`'s parser in debugging mode. This is primarily for development.

`-t` Use contents of file '`doctmp`' as a document template in which to enclose `PSTricks` output code. The code is inserted in place of the first instance of the escape string `%%SKETCH_OUTPUT%%`.

`-T` Causes `PSTricks` output to be enclosed in default US document template text. Option '`-Tu`' is a synonym. Option '`-Te`' causes the Euro standard document template to be used. A '`p`' appended to any of these options causes the respective default `PSTricks` document template to be printed to standard output. An appended '`P`' is a synonym. An appended '`T`' causes the the `TikZ/PGF` template to be printed. An appended '`L`' prints the LATEX version of the document template, a synonym for the default. A '`C`' prints the `ConTeXt` template.

`-o` Use '`output.tex`' as the output file. The default is standard output.

`-D` Define a tag for purposes of selecting definition alternatives. See Section 3.3 [Definitions], page 21. The definition applies for all input files that follow unless the tag is undefined with '`-U`'.

`input`*i*`.sk` Input files, read in the sequence they are given.

`-U` Un-define a tag for purposes of selecting definition alternatives.

6 Building and installing sketch

Sketch is so small that compiling by brute force is probably best. The following command ought to do the trick on any systems where gcc is installed. Make sure to first change current directories to the place where you have unpacked the sources.

```
gcc *.c -o sketch.exe -lm
```

The '.exe' at the end is necessary for Windows systems. Drop it if your system is some version of Unix. Other C compilers ought to work as just as well. For example,

```
cl *.c -o sketch.exe
```

is the correct command for many versions of MS Visual C. In the latest versions, Microsoft has deprecated the -o option and, by default, does not define the __STDC__ macro. This causes problems with some versions of flex, bison, lex, and yacc, which are used to create the sketch scanner and parser. It's nearly always possible to find a set of options that compiles with no errors or warnings, and this means sketch is *very* likely to work correctly. For example, the Visual C++ 2005 Express Edition compiler (available free of charge from the Microsoft web site), flex version 2.5.4, and bison version 2.1 build error-free with

```
cl -DYY_NEVER_INTERACTIVE=1 -Za -Ox -Fesketch.exe *.c
```

For purists, there is also a makefile compatible with GNU make and gcc. The command

```
make
```

will build the executable, including the rebuilding of the scanner and parser with flex and bison if you have changed 'sketch.l' or sketch.y respectively.

To build this document in all its myriad forms (assuming you have the necessary conversion programs on your system), use

```
make docs
```

The possibilities are listed in this following table.

Format	**Converter**	**Pictures**	**Description**
manual.info	makeinfo	'.txt'	GNU Info.
manual.dvi	texi2dvi	'.eps'	TeX typeset output.
manual.ps	texi2dvi,dvips	'.eps'	Postscript.
manual.pdf	texi2dvi	'.pdf'	Adobe PDF.
manual.html	makeinfo	'.png'	A single web page.
manual/index.html	makeinfo	'.png'	Linked web pages, one per node.

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