1 Prime Numbers

A *prime number* is a positive integer other than 1 that is only divisible by 1 and itself.

As you will show in Exercise 1.1, there are infinitely many primes. The number of primes that are smaller than a given natural number n is denoted $\pi(n)$.

Exercises

Exercise 1.1 (Euclid's Theorem). Show that there are infinitely many prime numbers.

Exercise 1.2. Find an asymptotic formula for $\pi(n)$. *Hint:* You might find Exercise 2.1 helpful.

2 Zeta function

The zeta function is given by $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$, where s is a complex number with real part bigger than 1. For example $\zeta(2) = \frac{\pi^2}{6}$.

Exercises

Exercise 2.1. Extend ζ as far as possible and find all zeros of the function.